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Summit Consulting, LLC

Declaration of Albert J. Lee, Ph.D.

12/19/2014

## **Introduction**

1. I have been retained by Quinn Emanuel Urquhart & Sullivan, LLP, counsel for Plaintiff Federal Housing Finance Agency (FHFA), as conservator for the Federal National Mortgage Association and the Federal Home Loan Mortgage Corporation, to evaluate various aspects of the estimations and implementation of the Greenfield Automated Valuation Model ("Greenfield AVM") in the FHFA cases.

Specifically, for the Nomura action, I have been asked to evaluate:

- i. The calculation of the 95 percent confidence intervals associated with individual Greenfield AVM forecasts;
- ii. The application of the forecast standard error ("FSD") in Dr. John Kilpatrick's analysis;
- iii. The appropriateness of the Greenfield AVM's cross-validation ("C.V.") filter and the sensitivity of average Greenfield AVM forecasts to various C.V. filter threshold settings;
- iv. The standard measure of predictive power of the Greenfield AVM;
- v. The comparison of Greenfield AVM forecasts against observed sale prices as an independent measure of the Greenfield AVM's predictive power;
- vi. The comparison of Greenfield AVM forecasts against the same forecasts by contemporaneous AVMs; and
- vii. Whether the Greenfield AVM forecast performance degrades for subject properties in California.

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### **Qualifications**

2. Since 2003, I have been Senior Economist and Founding Principal of Summit Consulting LLC, a consultancy specializing in econometric modeling and statistical sampling techniques. I received a Ph.D. in economics from the University of California at Los Angeles ("UCLA") in 1999, and a B.A. in economics and mathematics from the University of Southern California in 1992. I have published in the areas of economics and mathematics. I have taught undergraduate statistics, graduate advanced quantitative methods, and graduate econometrics at UCLA, George Washington University, and Columbia University, respectively. Since 2012, I have been certified as an Accredited Statistical Professional by the American Statistical Association. I am a member of the American Economic Association, American Statistical Association, National Association for Business Economics, Washington Statistical Society, and the Econometric Society.

3. I have estimated and developed econometric models for various federal agencies, including the U.S. Department of Housing and Urban Development, the U.S. Department of Labor, and the U.S. Department of Agriculture, as well as for litigation matters. Many of these models have an application in real estate finance. In addition to developing econometric models, I have extensive experience validating econometric models and verifying the accuracy of their forecasts. Under my direction, a team of analysts validated and verified econometric models for the U.S. Small Business Administration, the National Credit Union Administration, the Federal Deposit Insurance Corporation, and Freddie Mac. These engagements involved large and complex databases, and sophisticated econometric techniques.

4. My resume, including a list of my publications, is provided in Appendix A.

5. I base my opinions on my academic training and professional experience. My firm is compensated at a rate of \$569 per hour for my time. My compensation is not contingent upon my findings or the outcome of this case.

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### Summary of Conclusions

6. In connection with my preparation of this declaration, I reviewed the following materials concerning the Greenfield AVM and Dr. Kilpatrick's analysis:

- i. The Expert Report of John A. Kilpatrick, Ph.D., dated May 15, 2014;
- ii. The Expert Report of John A. Kilpatrick, Ph.D., dated October 6, 2014;
- iii. The Deposition of John A. Kilpatrick, dated November 13 and 14, 2014;
- iv. The Expert Report of John. A. Kilpatrick, Ph.D., dated June 27, 2014;
- v. The Expert Report of Jerry A. Hausman, dated August 14, 2014;
- vi. The Expert Report of Hans R. Isakson, dated August 14, 2014;
- vii. The Expert Report of Lee Kennedy, dated August 14, 2014; and
- viii. Computer programs of the Greenfield AVM and its outputs and variations on these programs produced by other experts.<sup>1</sup>

7. Based on this review, I reach the following conclusions.

- i. In his October 6, 2014 Expert Report, Dr. Kilpatrick calculated 95 percent confidence intervals for individual Greenfield AVM forecasts based on loan-level projection standard error.<sup>2</sup> These 95 percent confidence intervals are consistent with accepted statistical and scientific practices. Dr. Kilpatrick also presented 95 percent confidence intervals for individual Greenfield AVM forecasts in his expert report dated May 15, 2014.<sup>3</sup>
- ii. FSD quantifies the differences between an AVM's predictions and sales price. The Greenfield FSD of 15.1 percent was calculated based on a benchmarking validation exercise. It defines a

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<sup>1</sup> A full list of the materials I reviewed and relied upon in connection with this Declaration is collected in Appendix B.

<sup>2</sup> Expert Report of John A. Kilpatrick, Ph.D., dated October 6, 2014, pages 2-3.

<sup>3</sup> Expert Report of John A. Kilpatrick, Ph.D., dated May 15, 2014, Exhibit 6-2.

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benchmark to determine which subject properties were sent to the Credibility Assessment Model methodology (“CAM”) for further review. This application of FSD is non-statistical and had no bearing or impact on the Greenfield AVM estimates for the Nomura subject properties.

- iii. The C.V. filter eliminates statistical outliers when estimating the value of a subject property. Applying filters to eliminate outliers is a standard practice in econometric modeling, and is broadly supported by academic research and published articles.<sup>4</sup> Currently, the C.V. filter threshold is set at 0.25. I analyzed the sensitivity of the Greenfield AVM forecasts to a range of C.V. filter threshold settings, and I concluded that the average Greenfield AVM forecasts vary within a narrow range as the filter threshold is relaxed. In other words, the C.V. filter setting has a minimal and non-material impact on the average Greenfield AVM forecasts.
- iv. I examined the standard measure of the predictive power of a model, called the R-squared, for the Greenfield AVM regression models. The R-squared statistic measures how closely the model’s predictions are to the sales prices for the properties used in the model. Mathematically, the R-squared statistic quantifies the percentage of the variability in sales price explained by the Greenfield regression model. The R-squared statistic varies from 0 to 100 percent.<sup>5</sup> The average R-squared statistics are 85.6 percent for the OLS model and 86.6 percent for the OLSXY model, which are already close to the maximum of 100 percent. Therefore, introducing additional predictors to the Greenfield AVM cannot result in substantial improvement of the predictive power of the model.

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<sup>4</sup> Ruppert, David, and Raymond J. Carroll. "Trimmed least squares estimation in the linear model." *Journal of the American Statistical Association* 75, no. 372 (1980): 828-838.

<sup>5</sup> I use what is typically termed an “adjusted” R-square throughout, which adjusts the standard value downward according to the complexity of the model. There is no material difference between the R-square and the adjusted R-square in this application.

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- v. I examined how well the Greenfield AVM performed by comparing its forecasts to the observed sale price of subject properties in the Nomura, Goldman, HSBC, and Ally cases. On average, the Greenfield AVM forecasts were close to the sales price, but lower by an average of 0.04% percent – or four hundredths of a percent.<sup>6</sup>
- vi. I replicated Dr. Kilpatrick's comparison of the Greenfield AVM to four AVMs provided by Mr. Lee Kennedy in the Nomura case.<sup>7</sup> The Greenfield AVM forecasts produce, on average, higher valuations, which is favorable to the defendant insofar as it results in lower calculated inflation rates on average. Moreover, the Greenfield AVM forecasts exhibit lower overall sales variability as compared to the alternative AVMs proposed by Mr. Kennedy. Thus, the criticisms that the Greenfield AVM values are too low and favor the plaintiff, or are otherwise unreliable, are unsupported by data.
- vii. I then replicated Dr. Kilpatrick's comparison of the Greenfield AVM to three AVMs provided by Mr. Kennedy for the Goldman Sachs and HSBC cases. When considering the middle 90 percent of the Greenfield sales properties, on average, the Greenfield AVM 1) predicts higher AVM values compared to the commercial AVMs, which favors the defendant indicates that the Greenfield AVM results are conservative, and 2) has a lower standard deviation than the commercial AVMs, showing that the Greenfield AVM is more precise.
- viii. I next replicated Dr. Kilpatrick's comparison of the Greenfield AVM to Nomura's own contemporaneous AVMs that, as I understand, were utilized for due diligence. On average, the Greenfield AVM has a lower bias and a lower standard deviation than the Nomura AVM used for

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<sup>6</sup> Calculated as: (Greenfield AVM forecast - sale price)/sale price.

<sup>7</sup> Expert Report of Lee Kennedy, dated August 14, 2014.

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due diligence – facts which suggest that the Greenfield AVM is more reliable and accurate than those other AVMs.

- ix. I examined the effect of correcting the back-log transformation in the Greenfield AVM. This correction is brought up in the expert report of Dr. Jerry Hausman (“Hausman Report”). The correction methodology suggested in the Hausman Report, when performed properly, results in a less than a one percent increase to the average Greenfield AVM forecasts. The corrections performed in the Hausman Report, which incorrectly incorporate the error term (i.e. the difference between predicted value and sales price) for the observations that are rejected by the C.V. filter into the log transformation, are inconsistent with the standard method and inappropriate.
- x. I examined Greenfield AVM forecast performance by geography. Relative to sale price, I observed the Greenfield AVM produces *more* precise forecasts for subject properties in California than subject properties located in other states.

#### **Organization of this Declaration**

- 8. I organize my declaration into the following sections:
  - i. Section A discusses the Greenfield AVM’s statistical confidence intervals for subject properties.
  - ii. Section O discusses the derivation and application of the FSD derived from the validation gating benchmark.
  - iii. Section O discusses the appropriate use of filters to eliminate statistical outliers in econometric modeling and discusses the sound theoretical underpinning and application of the C.V. filter.
  - iv. Section G presents a summary of the standard statistical measure of predictive power for the Greenfield AVM.

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- v. Section H presents a comparison of the sale prices and the Greenfield AVM forecasts for subject properties.
- vi. Section O summarizes the changes to the average Greenfield AVM forecasts due to Dr. Hausman's inappropriate back-log transformation formula.
- vii. Section OI presents the performance of the AVM in California.
- viii. Appendix A contains my resume.
- ix. Appendix B contains a list of materials reviewed.

**A. Greenfield AVM Produces Loan-Level 95 Percent Confidence Intervals**

9. The Greenfield AVM is used in two separate but related processes: valuation and a benchmarking gating validation. To value a subject property, the Greenfield AVM implements the following steps:

- i. Property data are retrieved from the subject property's county, and are pared down to those properties that match the Greenfield AVM's criteria.<sup>8</sup>
- ii. The OLS and OLSXY regressions produce preliminary model statistics to implement the C.V. filtering process.

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<sup>8</sup> Property data were sourced from CoreLogic. Properties are always omitted from the Greenfield AVM if they are missing assessed value, sale date, or sale price; if they are a property type other than a single family residence or condo; if their sale date is outside of the 2002-2007 period; if they are not coded as arms'-length or grant deed transactions; or if they are one transaction in a multiple parcel sale. For each subject property, CoreLogic properties are omitted if their sale was executed more than one year before the subject property's transaction or any time after the subject property's transaction, or if they are of a different property type than the subject property. If there are more than 2,000 properties matched in the county, the 2,000 geographically closest properties are retained for regression analysis. If there are fewer than 100 properties, the subject property is marked "No Hit." For the "No Hit" subject properties, the Greenfield AVM does not produce a forecast.

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iii. The C.V. filter is applied by calculating the PRESS residual for each property within each model.<sup>9</sup> Properties to be used in the final model regressions are defined to be those for which the PRESS residual is less than 0.25.

iv. The OLS and OLSXY regressions are run using this final set of properties, and the individual subject property value forecast is calculated.

v. In cases where both the OLS and OLSXY produce a forecast, the two forecasts are averaged to generate the Greenfield AVM's forecast for the subject property.<sup>10</sup>

10. In Dr. Kilpatrick's Expert Report, dated October 6, 2014, he provides 95 percent confidence intervals for the Greenfield AVM forecasts based on loan-level prediction standard errors. The prediction standard error is a measure of the forecast precision of the model. Most software packages produce these standard errors as output to a regression model. The 95 percent confidence intervals can be constructed using standard errors. Because the Greenfield AVM values each subject property based on a separate regression model, standard errors and 95 confidence intervals vary from subject property to subject property.<sup>11</sup> This method of producing a 95 percent confidence interval, used by Dr. Kilpatrick in his Expert Report dated October 6, 2014, is statistically valid and consistent with sound scientific practice. I recalculated these intervals and verified that they are correct.

#### FSD Determines which Subject Properties Receive CAM Review

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<sup>9</sup> The PRESS residual is calculated as each county property's predicted forecast residual divided by one minus the county property's leverage. The PRESS residual is a "leave one out" statistic, and thus is calculated by taking the residual in the model *leaving out* the particular property for which the residual is calculated. For a specific formula, see: Thaddeus Tarpey, "A Note on the Prediction Sum of Squares Statistics for Restricted Least Squares," *The American Statistician* 54, no. 2 (2000): 116-118.

<sup>10</sup> It is possible only one of the two models will return a value, in which case the one returned value is used as the Greenfield AVM's estimated value.

<sup>11</sup> 95 percent confidence intervals are constructed using a value of 1.96 multiplied by the prediction standard error. See: Damodar Gujarati, *Basic Econometrics* (New York: McGraw-Hill, 1978), 91.



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11. To establish a gating benchmark FSD by which subject appraisals could be funneled to Dr.

Kilpatrick's credibility assessment methodology, the Greenfield AVM performs the following steps:

- i. The 25 million CoreLogic properties used during the valuation stage are randomly sub-divided into two groups, by county: one containing 90 percent of properties and one containing 10 percent of properties. Properties in the 10 percent set are treated as "meta-subject" properties. The Greenfield AVM uses the 90 percent set of properties to create regression models that value the 10 percent set of meta-subject properties.
- ii. To generate the gating FSD, the meta-subject properties are removed if either of two conditions is met:
  - a. The AVM valuation is more than twice the recorded sales price (i.e. "high forecast filter") or
  - b. The per-county sales-price-to-assessed-value (SPTAV) ratio is outside the middle 30th percentile (i.e. between the 35<sup>th</sup> percentile and the 65<sup>th</sup> percentile).
- iii. For the subject properties that meet these two conditions, the FSD is calculated as the standard deviation of the percentage difference between the filtered meta-subjects' forecasted sales prices and their observed sales price.<sup>12</sup> The standard deviation is a standard statistical measure of variability, and is calculated here by averaging the squared percentage difference between the AVM value and the sales price.<sup>13</sup> Based on this calculation, the FSD was determined to be 15.1 percent for the Greenfield AVM.

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<sup>12</sup> This is computed as the statistical standard deviation of (Greenfield AVM forecast - sale price)/sale price.

<sup>13</sup> This is a simplification of the calculation. For the specific formula, see: William H. Greene, *Econometric Analysis*. (New York: Macmillan, 1993), 56.

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12. Dr. Hausman indicated that the size of the FSD is an artifact of the two filters. He claimed that the FSD “explodes” to 2,728 if neither filter is applied.<sup>14</sup> However, he failed to point out that this FSD “explosion” is caused by only a tiny fraction of observation properties in the valuation gating benchmark exercise. For example, instead of eliminating the entire SPTAV filter, which is currently set at the middle 30 percent, I reset this filter at the middle 99 percent, which eliminates about one percent of the properties. This change alone causes the FSD to drop from 2,728 (when no filter is applied) to 56.6, (when one percent of the properties are removed). Alternatively, instead of eliminating the entire high-forecast-error filter, which is currently set at 100 percent, I reset this filter at 500 percent, which eliminates about 0.5 percent of the properties. This change alone causes the FSD to drop from 2,728 (when no filter is applied) to 0.477 (when approximately 0.5 percent of the properties are removed). This analysis underscores that a tiny fraction of the observation properties causes the so-called explosion. Dr. Hausman performed no investigation of these properties to determine if there were non-market transactions, non-arm’s-length transactions, or data recording errors that must be excluded.

13. Using the FSD, Dr. Kilpatrick identified subject properties with an appraised value at least 15.1 percent higher than their corresponding Greenfield AVM forecast. So identified, these subject properties were referred to the CAM for in-depth appraisal analysis. This application of the FSD is not the result of a statistical test and does not rely on a probability calculation. Rather, it was based on Dr. Kilpatrick’s decision that subject properties with an appraised value 15.1 percent higher than the Greenfield AVM forecasts’ gating benchmark established by his validation gating benchmark exercise should be subjected to review by the CAM.

Cross Validation (C.V.) Filter Improves the Reliability of Greenfield AVM Forecasts

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<sup>14</sup> Hausman Report, dated August 14, 2014, page 18.

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**B. Statistical Outliers Distort Regression Model Forecasts**

14. Regression models produce results that are akin to arithmetical averages.<sup>15</sup> Like any arithmetical average, regression models are sensitive to extreme values. Namely, the numerical results of a regression estimate can vary substantially depending on whether a single extreme value is included in the calculation. Such an outlier can unduly and inappropriately influence the numerical results of regression estimates if left untreated.

15. Outliers are not limited to simple errors in this context, but instead include observations that disproportionately influence the model estimation and forecasts.<sup>16,17</sup> The following example is instructive. Suppose we want to forecast income based on educational attainment for American adults using a regression model. In particular, we wish to quantify the impact of a college degree on income. On average, one would expect higher educational attainment (e.g. the attainment of a college degree) to correlate with a higher level of income. Bill Gates earns a very high income yet is a college dropout. While exceptionally high, his income is not a data error. However, by including him as a data point when estimating a regression model that includes educational attainment as the *only* predictor, the relationship between the predictor and the outcome would be distorted for other American adults.

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<sup>15</sup> Arthur Goldberger, *A Course in Econometrics*, (Cambridge, MA: Harvard University Press, 1991), 8-9, 46-49.

<sup>16</sup> From Frank Harrell, *Regression Modeling Strategies: With Applications to Linear Models, Logistic Regression, and Survival Analysis*, (New York: Springer, 2001), 74.: "Extreme values of the predictor variables can have a great impact, even when these values are validated for accuracy. Sometimes the analyst may deem a subject so atypical of other subjects in the study that deletion of the case is warranted."

<sup>17</sup> From R.D. Cook and S. Weisber, "Residuals and Influence in Regression." in *Monographs on Statistics and Applied Probability*, ed. D.R. Cox and D.V. Hindkley (New York: Chapman and Hall, 1982), 1.: "It is well known, for example, that inferences based on ordinary least squares regression can be strongly influenced by only a few cases in the data, and the fitted model may reflect unusual features of those cases rather than the overall relationship between the variables."

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That is, a model including Gates as an observation might depress the model's estimation of the effect of educational attainment on income.<sup>18</sup>

16. Bill Gates might not have been an outlier if we had a reliable measure of other qualities that lead to high income, which are not included in the model (e.g. intelligence or talents, which are not readily observable). In other words, statistical outliers are best understood given the context of a specific model based on available predictors. The filter used by Dr. Kilpatrick determines outliers by accounting for precisely those predictors that are in the model, and does so appropriately.

### C. C.V. Filter is Based on PRESS Statistics

17. Consequently, the relevant questions have always been how to identify outliers, and how many of them to remove.<sup>19</sup> The applied literature suggests a class of measures based on the concept of *leverage*. Observations that exert a high degree of leverage change the model enough that their removal may be necessary.<sup>20</sup> Statistical measures used to determine outliers and their effects include PRESS residuals<sup>21</sup> and Cook's Distance. Dr. Kilpatrick's C.V. filter is based on the PRESS residual.

Conceptually, the PRESS residual is the difference between the sales price of a property and the AVM

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<sup>18</sup> In contrast, the inclusion of an individual *with* a college degree and extremely high income would tend to over-estimate the general effect of education on income. In other words, outliers, even if correct, can be a problem.

<sup>19</sup> Examples of excluding outliers include: Jerry Hausman, Gregory K. Leonard, and J. Gregory Sidak, "Does Bell Company Entry Into Long-Distance Telecommunications Benefit Consumers?" *Antitrust Law Journal* 70, no. 2 (2002): 471; Martin Burda, Matthew Harding, and Jerry Hausman, "A Bayesian Semiparametric Competing Risk Model with Unobserved Heterogeneity," *Journal of Applied Econometrics* (2014): 19; Jerry Hausman and Ephraim Leibtag, "Consumer Benefits from Increased Competition in Shopping Outlets: Measuring the Effect of Wal-Mart," *Journal of Applied Econometrics* 22, no. 7 (2007): 1174.

<sup>20</sup> Peter Kennedy, *A Guide to Econometrics*. (Cambridge, MA: MIT Press, 2003), 379.

<sup>21</sup> Thaddeus Tarpey, "A Note on the Prediction Sum of Squares Statistics for Restricted Least Squares." *The American Statistician* 54, no. 2 (2000): 116-118.

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estimate of that property, provided that property is not used to create the model.<sup>22</sup> In the Bill Gates example, we would predict his earnings by averaging the earnings of college dropouts, besides Gates himself, and then observe the difference between his predicted and actual incomes. We would not expect our model to predict Bill Gates's income well and thus we would expect to find a high PRESS residual. In the same way that including Gates in the model would inappropriately alter the model's estimation of the relationship between earnings and education attainment, including properties with high PRESS residuals would substantially (and improperly) change the Greenfield AVM's estimation of the relationship between a property's value and its predictive characteristics.

#### **D. The Use of the C.V. Filter is Not Data Snooping**

18. Removing outliers is a common practice in applied econometric modeling, especially when using raw data that are not model-ready (e.g., Corelogic data).<sup>23</sup> Regression techniques routinely remove outliers. For example, Robust Regression in Stata, a professional standard statistical software program, states the following in its reference manual: "first performs an initial screening based on Cook's distance > 1 to eliminate gross outliers..."<sup>24</sup> The C.V. filter serves a similar function.

19. Although literature indicates that the use of the PRESS residual is appropriate, the amount of filtering used is left to the experience and judgment of the model developer and user.<sup>25</sup> In this case, the

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<sup>22</sup> Mathematically, the PRESS residual is the "ordinary residual weighted according to the diagonal elements of the hat matrix  $h_{ii}$ ". D. C. Montgomery, E. A. Peck, and G. G. Vining, "Introduction to Linear Regression Analysis," 3rd ed. (New York: John Wiley & Sons, 2003): 134.

<sup>23</sup> The necessity of removal of observations from applied regression models has long been acknowledged. From Cook, R. Dennis Cook, "Detection of Influential Observation in Linear Regression," *Technometrics*, Vol. 19, Nono. 1 (1977): 15: "It is perhaps a universally held opinion that the overall summary statistics (e.g.,  $R^2$ , B)[...] arising from data analyses based on full rank linear regression models can present a distorted and misleading picture."

<sup>24</sup> See StataCorp. *Stata 11 Base Reference Manual*. College Station, TX: Stata Press, 2009, page 1646.

<sup>25</sup> From R. Dennis Cook, and Sanford Weisber. "Residuals and Influence in Regression." in *Monographs on Statistics and Applied Probability*, ed. D.R. Cox and D.V. Hindkley. (New York: Chapman and Hall, 1982), 33: "The action to be taken as a result of finding an outlier, such as case deletion or downweighting, will depend on the context of the problem at hand."

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Greenfield AVM sets the C.V. filter at 0.25 in the Nomura case.<sup>26</sup> It is notable that the 0.25 filter setting has been consistently applied in the Greenfield AVM for previous cases, for over 7,000 subject properties, and over 14,000 regressions for the Nomura, Goldman Sachs, HSBC, and Ally cases. The consistent application of the same filter setting over a large number of regressions defies any suggestion of “data snooping,” which would require regression-by-regression inspections and adjustment.

#### **E. The Average Greenfield AVM Forecasts are Stable to Various Filter Settings**

20. I further assessed the sensitivity of the Greenfield AVM forecasts to the 0.25 C.V. filter threshold. To do this, I recalculated the Greenfield AVM forecasts and other model statistics using a range of thresholds that filtered fewer properties than the 0.25 threshold setting. I found the threshold setting has a limited impact on the average Greenfield AVM forecasts.

21. Table 1 tabulates the average Greenfield AVM forecasts, the average percentage differences between the appraisal values and the Greenfield AVM forecasts, the R-squared values, and the percentage of properties included under different C.V. filter threshold settings (from 0.25 to the complete removal of the filter).

22. Table 1 shows that:

- i. Set at 0.25, the C.V. filter removes about 22 percent of the county properties. These are properties that are (1) not the result of arm’s length transactions, and/or (2) the result of errant data, and/or (3) situations where significant hedonic characteristics of the property are not discernible from available data, making the properties’ inclusion analogous to the Bill Gates example discussed above.

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<sup>26</sup> Because the residual is in log terms, this amounts to a residual of about, but not exactly, 25% more or less than the property’s value.

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- ii. The average Greenfield AVM forecasts fluctuate within a range of about \$5,000, less than two percent of the average value (< 2%), and reach a maximum of \$253,052 (when the C.V. filter threshold is set at 0.3) and a minimum of \$248,629 (when the C.V. filter is removed completely). This indicates that the effect of relaxing the C.V. filter on the Greenfield AVM forecasts is not material and does not move in a single direction.
- iii. Based on my analysis, the average R-squared values for the OLS and OLSXY models decrease but are consistently over 70 percent until the filter is completely removed, indicating that the Greenfield AVM is a consistently strong model that is insensitive to the C.V. filter threshold settings.

Table 1: Average Greenfield AVM Outputs by CV Filter Threshold Setting

CV Filter Threshold	AVM Forecast	Appraisal-to-AVM Difference Ratio	OLS Regression R Squared	OLSXY Regression R Squared	Properties Included in the OLS Regression	Properties Included in the OLSXY Regression
0.25	\$ 253,005	8.92%	86%	87%	78%	79%
0.3	\$ 253,052	8.69%	83%	85%	83%	83%
0.4	\$ 252,602	8.57%	80%	82%	89%	89%
0.5	\$ 252,247	8.59%	78%	79%	92%	93%
0.6	\$ 252,326	8.71%	76%	77%	94%	95%
0.7	\$ 251,894	9.00%	75%	76%	96%	96%
0.8	\$ 252,010	9.20%	73%	74%	96%	97%
0.9	\$ 252,447	9.35%	72%	73%	97%	97%
1	\$ 252,474	9.46%	70%	72%	98%	98%
None	\$ 248,629	12.42%	54%	56%	100%	100%

23. Collectively, this sensitivity analysis shows that the average Greenfield AVM forecasts are stable to various filter settings. Only two percent of the county properties in the OLS regressions are responsible for the largest model performance degradation from an R-squared of 70 percent (when the C.V. filter is set at 1) to an R-square of 54 percent (when the C.V. filter is completely removed).

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**F. C.V. Filter Eliminates Extreme Sale Prices Relative to Assessed Values**

24. By progressively changing the C.V. filter threshold setting, I conclude that the C.V. filter serves to eliminate comparable properties with extreme sale prices relative to assessed values. Table 2 shows that the average sale price for the excluded comparable properties in the OLS regressions increases progressively as the C.V. filter setting changes from 0.25 to 1.00. At its current setting of 0.25, 398 comparable properties are excluded on average. The average sale price for these excluded properties is about \$474,982. As the filter threshold setting increases, it excludes fewer properties. If the C.V. filter were set at 0.3, it would exclude 306 properties. However, these 306 excluded properties have an average sale price of \$500,393, or about 5% higher than the 398 properties that are currently excluded. This analysis indicates that the C.V. filter progressively removes more extreme properties, relative to value-related characteristics.

25. The C.V. filter does not filter on sales price alone. It also eliminates properties with extreme SPTAV ratios. At its current setting of 0.25, the SPTAV ratio for the excluded properties is about 2.57 ( $=\$474,982/\$184,988$ ). This compares to a SPTAV ratio of 1.90 ( $=\$299,119/\$157,772$ ) for the included properties. As the C.V. filter setting increases, fewer properties are excluded. I note that the remaining excluded properties have an increasingly extreme SPTAV ratio, from 2.65 (C.V.=0.3) to 4.74 (C.V.=1.00). This compares to the included properties of 1.89 (C.V.=0.3) to 1.92 (C.V.=1.00).

26. Based on these two observations, I conclude that the C.V. filter eliminates properties (1) with extreme sale prices, and (2) with extreme SPTAV ratio. The C.V. filter is therefore an effective tool for eliminating outlier, extreme properties among a large population.

Table 2: Comparing Comparable Properties Included and Excluded by C.V. Filter Thresholds



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OLS Model		Excluded Comparables					Included Comparables				
Cross Validation Parameter	Count of Potential Comparables	Count of Comparables Excluded	Percent of Comparables Excluded	Excluded Comparables Sales Price	Excluded Comparables Assessed Value	Sale Price to Assessed Value Ratio	Count of Comparables Included	Percent of Comparables Included	Included Comparables Sales Price	Included Comparables Appraisal Value	Sale Price to Assessed Value Ratio
0.25	1,836	398	21.7%	\$ 474,982	\$ 184,988	2.57	1,439	78.3%	\$ 299,119	\$ 157,772	1.90
0.30	1,829	306	16.7%	\$ 500,393	\$ 189,119	2.65	1,523	83.3%	\$ 298,745	\$ 158,417	1.89
0.40	1,827	197	10.8%	\$ 565,847	\$ 196,288	2.88	1,630	89.2%	\$ 294,085	\$ 159,387	1.85
0.50	1,824	137	7.5%	\$ 643,495	\$ 204,719	3.14	1,687	92.5%	\$ 290,952	\$ 160,109	1.82
0.60	1,824	102	5.6%	\$ 726,514	\$ 210,822	3.45	1,722	94.4%	\$ 296,825	\$ 160,638	1.85
0.70	1,824	79	4.3%	\$ 818,926	\$ 217,029	3.77	1,745	95.7%	\$ 303,060	\$ 160,979	1.88
0.80	1,824	64	3.5%	\$ 929,801	\$ 224,666	4.14	1,760	96.5%	\$ 305,950	\$ 161,280	1.90
0.90	1,824	53	2.9%	\$ 1,028,383	\$ 232,488	4.42	1,771	97.1%	\$ 307,954	\$ 161,478	1.91
1.00	1,824	45	2.5%	\$ 1,114,933	\$ 235,242	4.74	1,779	97.5%	\$ 309,972	\$ 161,666	1.92
None	1,824						1,824	100.0%	\$ 387,563	\$ 163,666	2.37

### G. The Predictive Power of the Greenfield AVM is High

27. The primary measure of a model's predictive power is called R-squared. By way of background, a regression model is a more sophisticated version of averages. Rather than just assuming a property's sales price will be a simple average of sales price of comparable properties, a regression model calculates *conditional* average of sales price, based on a number of characteristics (or predictors). These predictors may be purely hedonic, like square footage, or a substitute for one or more hedonic qualities, like tax assessed value. The predictors are said to have explanatory power if the resultant conditional average is sufficiently different from a simple average.

28. The R-squared of a regression measures how close the conditional average is to the sales price as compared to the simple average of sales prices. In the context of the Greenfield AVM, R-squared statistics quantify the percentage of statistical variance that is explained by the Greenfield AVM.<sup>27</sup> The maximum potential R-squared statistic is 100 percent, implying a perfect model.<sup>28</sup> The minimum R-squared is 0, implying that using the predictors to explain the outcome is no better than simply estimating the county average for every property in a county.

<sup>27</sup> N.J.D. Nagalkerke, "A note on a general definition of the coefficient of determination." *Biometrika* 78, no. 3 (1991): 691-692.

<sup>28</sup> William H. Greene, *Econometric Analysis*. (New York: Macmillan, 1993), 191-195.

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29. I calculated the R-squared statistic for all Nomura subject properties and found that the average is 86 percent.<sup>29</sup> Given the theoretical maximum of 100 percent, it is far from clear that introducing additional variables or eliminating presently-used variables would substantially improve the fit of the model.

30. In addition to the overall fit of the model as quantified by R-squared, we can also examine the regression coefficients associated with individual predictors. If these regression coefficients are statistically significantly different from zero, the associated predictors statistically contribute to the Greenfield AVM forecasts (i.e. the forecasts improve when the model contains those predictors). Consequently, these significant statistics are useful to quantify the importance of individual predictors to the forecasts.

31. The Greenfield AVM predicts sale prices based on available information. The strength of correlation between sale prices and predictors is central to the model's predictive power. The fact that sale prices could pre-date some of the predictors does not adversely impact the Greenfield AVM's predictive performance so long as the strength of the correlation is unaffected by the timing.

32. In my analysis, I observed that the regression coefficients associated with tax assessed value are statistically significant in more than 99 percent of Greenfield AVM regressions.<sup>30</sup> I therefore conclude that even post-dated tax assessed value by itself statistically contributes to the forecasts.<sup>31</sup> This means that, for purposes of the Greenfield AVM, the relationship between tax assessed value and sales price is

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<sup>29</sup> The R-squared is 85.6 percent for the OLS model and 86.6 percent for the OLSXY model.

<sup>30</sup> Tax assessed value is an important predictor of market value. From William Goolsby, "Assessment Error in the Valuation of Owner-Occupied Housing," *Journal of Real Estate Research*, 13, no. 1 (1997), 43: "Assessed value provides important information about market value. It provides an informed opinion about the value of the property from a professional analysis. It is based on an on-site evaluation by trained professionals and should provide a better estimation of value than the standard hedonic equation approach."

<sup>31</sup> I judged this based on either one of the regression models (OLS or OLSXY).

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stable and consistent such that the use of tax assessed value from subsequent years to the year in which the property's value is being estimated does not degrade the Greenfield AVM's predictions due to the use of ex post data.

#### **H. Greenfield AVM Forecasts are as Precise as Other AVM Forecasts**

33. By definition, the R-squared is a measure of how well the model prediction comports with the data used to create the model. To externally validate the model, I replicated the analysis performed by Dr. Kilpatrick,<sup>32</sup> comparing his Greenfield AVM forecasts for the subject properties to those of the other "commercially available" AVMs utilized by Mr. Kennedy. I conclude that the Greenfield AVM performs well in valuing subject properties that were sold.

34. I verified Dr. Kilpatrick's findings in his Expert Report, dated June 27, 2014, which indicate that the Greenfield AVM performs exceptionally well (according to industry standards) for the Goldman Sachs, Ally, and HSBC subject properties. For each of these cases, the average sales error is 2.3% or lower (absolute value), and the standard deviation is 0.15 or lower, meeting the standards referenced in Dr. Kilpatrick's reports. This means that the Greenfield performs in line (i.e. within .15 ) with Dr. Kilpatrick's expectations based on his validation gating benchmark when applied to subject sales properties across prior FHFA cases. I also verified Dr. Kilpatrick's findings in his Expert Report, dated October 6, 2014, which indicates that the Greenfield AVM's performance remains within the industry standards for Nomura subject properties, with an average sales error of 1.26% and a standard deviation of 0.15 when analyzing the middle 90 percent of Nomura subject sale properties.<sup>33</sup>

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<sup>32</sup> Expert Report of John A. Kilpatrick, Ph.D., dated October 6, 2014, pages 8-9.

<sup>33</sup> Expert Report of John A. Kilpatrick, Ph.D., dated October 6, 2014, page 9.

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35. In terms of precision, I concluded that the Greenfield AVM performs similarly to the four AVMs utilized by Mr. Kennedy for the Nomura case. Recognizing outliers and atypical observations that could skew my analysis, I compared the AVM forecasts with sale prices for the middle 90 percent of properties (by excluding the top and bottom five percent of the sale prices based on median sales error).<sup>34</sup> The Greenfield AVM's sale error distribution is nearer to zero than the sale error distributions based on Mr. Kennedy's AVMs. In other words, excluding outliers, the Greenfield AVM forecasts exhibit the lowest bias as compared to other AVMs' forecasts. Moreover, using the middle 90% of values, the Greenfield AVM produces an error standard deviation that is lower than the four AVMs utilized by Mr. Kennedy. In other words, in terms of reliability, when using subject sales prices as a measure of market value, the Greenfield AVM outperforms all four of the AVMs utilized by Mr. Kennedy in his analysis. The Greenfield AVM performs comparably to the other commercial AVMs by removing even a single property, which according to Dr. Kilpatrick was a miscoded multi-parcel sales.<sup>35</sup>

36. In addition to verifying Dr. Kilpatrick's comparison between the Greenfield AVM forecasts and the other four commercially available AVMs, I made a similar comparison with Nomura's AVM used for the defendant's due diligence process. When comparing AVM performance on the same set of subject properties, I conclude that the Greenfield AVM outperforms the Nomura AVM in two dimensions. First, relative to the Nomura AVM, Greenfield AVM forecasts exhibit a lower level of bias (-0.30% vs -0.90%). Second, based on standard deviation, the Greenfield AVM forecasts are more precise than those of the Nomura AVM (0.150 vs 0.223). As Table 3 shows, compared to the Nomura AVM, the Greenfield AVM is less biased and more precise.

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<sup>34</sup> Averages and standard deviations are sensitive to outliers. Truncations by eliminating the extreme five percent at both end of a distribution could improve the reliability of these statistics. See National Research Council of the National Academies. *Reference Manual on Scientific Evidence, Third Edition*. Washington, D.C.: The National Academies Press, 2011, page 240.

<sup>35</sup> Expert Report of John A. Kilpatrick, Ph.D., dated October 6, 2014, pages 6-7.

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Table 3: AVM Sales Error Comparison for Properties Valued by All AVMs

Comparing Sales Error: Only For Loans Benchmarked by Every AVM						
Sales Error: (Sales Price - AVM)/ Sales Price	DQAE	DQP	Collateral Analytics	Real Info	Nomura	Greenfield
Count	85	85	85	85	85	85
Average	3.10%	3.60%	3.80%	4.80%	-0.90%	-0.30%
Mean Absolute Error	9.70%	9.50%	11.20%	13.90%	13.20%	11.40%
Median	1.90%	2.90%	4.20%	5.10%	-0.40%	0.20%
Median Absolute Error	6.40%	6.40%	7.60%	12.00%	6.40%	7.90%
Standard Deviation	0.138	0.132	0.151	0.175	0.223	0.150

37. I then performed the same sales validation analysis for the three AVMs utilized by Mr. Kennedy for the Goldman Sachs and HSBC cases (Collateral Analytics, Real Info, and RELAR). As Tables 4 and 5 show, comparing AVM forecasts with sale prices within the middle 90 percent (by excluding the top and bottom five percent of the sale prices based on median sales error), the Greenfield AVM is conservative, in that it produces higher predictions relative to Mr. Kennedy's AVMs. Moreover, using the middle 90% of values, the Greenfield AVM produces a standard deviation that is lower than the three AVMs utilized by Mr. Kennedy. Again, in terms of reliability, when using subject sale prices as a measure of market value, the Greenfield AVM outperforms the AVMs utilized by Mr. Kennedy in his analysis.

Table 4: AVM Sales Error Comparison for Goldman Sachs AVMs

Sale Error: (Sales Price - AVM)/Sales Price	Greenfield AVM Mid 90	Collateral Analytics AVM	Real Info AVM	RELAR AVM
Properties Valued	1,024	1,061	820	996
Average Sales Error	-1.9%	0.3%	1.7%	1.9%
Median Sales Error	0.0%	2.7%	3.1%	4.7%
Standard Deviation of Sales Error	0.155	0.225	0.202	0.426

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Table 5: AVM Sales Error Comparison for HSBC AVMs

Sale Error: (Sales Price - AVM)/Sales Price	Greenfield AVM Mid 90	Collateral Analytics AVM	Real Info AVM	RELAR AVM
Properties Valued	520	530	396	438
Average Sales Error	-2.3%	0.2%	0.3%	-0.1%
Median Sales Error	-1.1%	2.0%	2.3%	3.1%
Standard Deviation of Sales Error	0.143	0.206	0.185	0.240

**I. The Alternative Back-Log Transformation Imparts a Less than One Percent Change to the Average Greenfield AVM Forecast**

38. The Greenfield AVM estimates the log sale price of the subject property, and converts the estimates into dollars by an exponentiation.<sup>36</sup> This transformation, although appropriate for a median estimate,<sup>37</sup> may be adjusted by a factor in order to estimate the mean value. Dr. Kilpatrick quantified the magnitude of change due to this factor in his October 6, 2014 Expert Report.<sup>38</sup> I have reviewed his analysis, and I agree with his conclusion that the average change to subject property valuations owing to this factor is approximately 0.60 percent.

39. I also agree with Dr. Kilpatrick's determination that the Hausman Report contains a flawed and improperly biased analysis of this factor, and thus substantially exaggerates its impact. In particular, the Hausman Report inconsistently applies the Greenfield AVM in concluding that the Greenfield AVM forecasts underestimate by "an average of approximately \$39,500 per property or, in percentage terms, by approximately 18.0%".<sup>39</sup> On one hand, the Hausman Report applied the C.V. filter for estimation. On

<sup>36</sup> Exponentiation of a quantity is the process of raising that quantity to the power of another number, in this case the natural constant e (equal to about 2.7).

<sup>37</sup> Alastair J. Scott and Michael J. Symons, "A Note on Shortest Prediction Intervals for Log-Linear Regression," *Technometrics*, 13, no. 4 (1971): 889-894.

<sup>38</sup> Expert Report of John A. Kilpatrick, Ph.D., dated October 6, 2014, pages 7-8.

<sup>39</sup> Hausman Report, dated August 14, 2014, pages 21-23.

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the other hand, inconsistent to the estimation step, the Hausman Report improperly *removed* the C.V. filter completely in calculating the size of the correction factor. In other words, the Hausman Report used one set of data for the primary estimate, and a second set of data for the determination of the correction factor. In effect, Dr. Hausman's purported correction would force the Greenfield AVM to use atypical properties, based on extreme sales prices and sales price to assessed value ratios, as the basis for valuing the Nomura sample properties. The artificial influence of these extreme properties accounts for the average shift in value of 18.0% that Dr. Hausman reports.

40. This inconsistent treatment is problematic and results in an incorrect adjustment. First, the model estimation and the calculation of the factor are now disconnected, as they are based on two different datasets. Second, as shown earlier, the complete removal of the C.V. filter introduces statistical outliers (including sales properties with values as high as \$203 million), which serve to inflate the size of the factor. For these two reasons, the back-log transformation calculations performed in the Hausman Report are inapplicable and incorrect. As noted above, when I recalculated the impact of the back-log calculation, I determined that the change resulted in a less than one percent increase per subject property on average.

**J. Greenfield AVM Forecast Performance Does Not Degrade for Subject Properties in California**

41. In his report, Dr. Isakson ("Isakson Report") hypothesized that the Greenfield AVM forecast performance would degrade for subject properties in California because according to Dr. Isakson, Proposition 13 in California artificially depresses tax assessed values for subject properties in California.<sup>40</sup>

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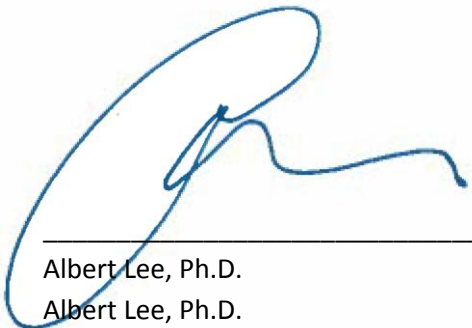
<sup>40</sup> Isakson Report, dated August 14, 2014, pages 40-41.

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42. I evaluated the impact of these potential issues by using sale prices from purchased subject properties as benchmarks. Specifically, I calculated the percentage differences between Greenfield AVM forecasts and sale price (sale error) as a measure of precision. I compared this measure of precision between purchase subject properties in California and those from other states. Table 6 shows that the Greenfield AVM forecasts are closer to the sale price for the 29 California subject properties than the 276 subject properties from other states. This analysis thus shows that performance does not degrade in California with respect to the relevant properties for which data are available (Nomura's subject properties that were associated with sales transactions).

Table 6: AVM Performance in California Compared to All Other States

(Sales Price - AVM) / AVM	California	Other States
Number of Loans	29	276
Average	0.94%	4.35%
Mean Absolute Error	9.03%	16.13%
Median	1.48%	2.65%
Median Absolute Error	6.66%	11.75%
SD	11.78%	20.94%




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Albert Lee, Ph.D.  
Albert Lee, Ph.D.



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Curriculum Vitae

### **Education/Certifications**

Ph.D. in economics, University of California, Los Angeles, 1999

M.A. in economics, University of California, Los Angeles, 1996

B.A. in economics and mathematics (cum laude), University of Southern California, 1992

ASA Accredited Statistical Professional (2012)

### **Relevant Experience**

Founding Principal and Lead Economist, Summit Consulting, LLC

Senior Consultant, Bates White (Ballentine) LLC

Manager, KPMG Quantitative Analysis Group

### **Academic Appointments**

Adjunct Associate Professor, School of International and Public Affairs, Columbia University, spring 2013

Adjunct Assistant Professor, Department of International Public Affairs, Columbia University, spring 2004

Visiting Assistant Professor, Department of Legislative Affairs, George Washington University, fall 2003

Visiting Assistant Professor, Department of Statistics, UCLA, 1999

### **Publications**

*Taxation, Growth and Fiscal Institutions: A Political and Economic Analysis*, Springer 2011.

"Testing the Double-Trigger Hypothesis Using Loan-Level Annual Financial Statement Data from an FHA-Insured Multifamily Program," under revision at *Real Estate Economics* (with Yvon Pho and Colin Cushman).

"Strategic Defaults in Commercial Mortgages with a Cash-in-advance Constraint: A Dynamic Programming Analysis," Working Paper (with Kenneth A. Baerenklau), 2011.

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“Law and Statistics of Combining Categories: Wal-Mart and Employment Discrimination Cases,” with Alan Salzberg, Proceedings of the 2010 Joint Statistical Meetings of the American Statistical Association, 2010.

“Why Settle? A Simulated Model of Strategic Interactions between Plaintiff Attorneys and Defendants in a Litigation Environment,” Working Paper, 2003.

“Anticipation, Worry and the Endogenous Determination of Risk Preference,” Working Paper (with Luis Locay), 2003.

“Classifying Rank 2 Quadratic Lattices by their Representation,” *Nova Journal of Algebra and Geometry* 2 (1993): 210-217 (with D. Estes).

### **Professional Membership**

American Economic Association

American Statistical Association

Econometric Society

National Association for Business Economics

Washington Statistical Society

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#### Materials Reviewed

In preparing this declaration, I have studied the Plaintiff's allegations and the Defendant's responses. Such materials that I reviewed or instructed researchers working under my direction to review include:

- Expert Report Concerning Accuracy of Appraisals of John A. Kilpatrick, 05/15/2014
- Expert Report of John A. Kilpatrick, 06/27/2014
- Expert Report of John A. Kilpatrick, 10/06/2014
- Deposition of John A. Kilpatrick, 11/13/2014 and 11/14/2014
- Expert Report of Jerry A. Hausman, 08/14/2014
- Expert Report of Hans R. Isakson, 8/14/2014
- Expert Report of Lee Kennedy, 08/14/2014
- Background materials from Expert Report Concerning Accuracy of Appraisals of John A. Kilpatrick, 05/15/2014
- Background materials from Expert Report of John A. Kilpatrick, 06/27/2014
- Background materials from Expert Report of John A. Kilpatrick, 10/06/2014
- Background materials from Expert Report of Jerry A. Hausman, 08/14/2014

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